**Homework 5**

**P16.1.3** Given the two functions: (a) *f*(*t*) = cos(100*πt*)sin(200*πt*) (b)  Show that  and  are periodic, determine their periods, and derive their FSEs.

**Solution:** (a) Using the trigonometric identity, cos(100*πt*)sin(200*πt*) = 0.5sin(100*πt*) + 0.5sin(300*πt*). This is an FSE of a periodic function having a period 2*π* /100*π* = 1/50 s ≡ 20 ms.

(b) =  = 0.25 + 0.25cos(200*πt*) – 0.25cos(400*πt*) – 0.25cos(200*πt*)cos(400*πt*). This last term is – 0.125cos(200*πt*) – 0.125cos(600*πt*) = 0.25 + 0.125cos(200*πt*) – 0.25cos(400*πt*) – 0.125cos(600*πt*). This is an FSE of a periodic function having a period 2*π* /200*π* = 1/100 s ≡ 10 ms.



**P16.1.6** A periodic voltage is represented by the expansion:  and has the magnitude spectrum shown in Figure P16.1.6. Determine the average value and the fundamental frequency of *v*(*t*).

**Solution:** From the figure, the ordinate at n = 0 = 5 and is the average value. From the exponent, *ω*0 = 200*π*, so that *f*0 = 200*π*/2*π* = 100 Hz.



**P16.1.10** Given the periodic function *f*(*t*) shown in Figure 16.1.10. Determine the amplitude of the third harmonic.

**Solution:** The function is odd and half-wave symmetric, so that: , where *T* = 8 and *ω*0 = 2*π*/*T* = *π*/4. Hence,  . The amplitude is therefore  V.

**P16.1.16** (a) Derive the first three terms of the trigonometric FSE of the periodic function *i*(*t*) A shown in Figure P16.1.6; (b) repeat (a) when *i*(*t*) is advanced or delayed by 2 s.



**Solution:** (a) The function is even, *bn* = 0 for all *n*, *a*0 =, and *ω*0 =.  += + ; sin*nπ* = 0 for all integer *n* and the sin*nπ/2* terms cancel out. It follows that *an* = , *a*1 = and *a*2 =. The first three terms of the trigonometric FSE are: *i*(*t*) = 1.5 = 1.5 – 0.8106cos*πt*/2 – 0.4053cos*πt* A.

(b) When the function is advanced or delayed by 2 s, *t* is replaced by (*t* ± 2) or the phase angle is changed by ±2*nω*0 = ±*nπ; i*(*t*) *=* 1.5 = 1.5 A.

**P16.1.17** Given a full-wave rectified waveform of period *T* as shown in Figure 16.4.1b, except that because of dissymmetry in the rectifier circuit, the half-sinusoids are not all of the same amplitude but alternate with amplitudes of 12 V and 10 V. Derive the FSE, assuming that the sinusoid centered at the origin has a an amplitude of 12 V.



**Solution:** From Equation 16.4.2, the FSE of a half-wave rectified wave form of amplitude 12 centered at the origin is: cos*ω*0*t* + (cos2*ω*o*t* - cos4*ω*o*t*  + cos6*ω*o*t* + …). The FSE of a half-wave rectified waveform of amplitude 10 delayed, or advanced, by , where  = *π*, is:  cos(*ω*0*t* ± *π*) +

. The sum of the two is cos*ω*0*t* + cos2*ω*0*t* – cos4*ω*0*t* + cos6*ω*0*t* + … + cos2*nω*0*t* + … *n* = 1, 2, 3, …

**P16.1.20** Determine the magnitude and phase angle of the fundamental component of the periodic function *f*(*t*) shown in Figure P16.1.20, where *f*(*t*) = sin(2*πt*), for 0 ≤ *t* ≤ 0.25 s, and *f*(*t*) = 0.5sin(2*πt*), for 0.25 ≤ *t* ≤ 0.5 s.



**Solution:** The function is odd and of zero average with *ω*0 = 2*π* rad/s. Hence *an* = 0 for *n* ≥ 0 and . The integral is:  = 

;

*b*1 = . The amplitude is therefore 0.75, the phase angle being zero.

**P16.1.21** Derive the FSE of the periodic function shown in Figure P16.1.21, defined as:



*f*(*t*) = cos(*t* + *π*) – 2, -*π* < *t* < -*π*/2

*f*(*t*) = -cos*t* + 3, -*π*/2 < *t* < +*π*/2

*f*(*t*) = cos(*t* – *π*) – 2, *π*/2 < *t* < *π*

**Solution:** The function has an average component. Over half a period, from 0 to *π*, the difference between the positive and negative areas is 1×*π*/2. Divided by half a period, the average value is 1/2. Alternatively,

= == =.

If 0.5 is subtracted from the function, the function becomes quarter-wave symmetric, even, and of zero average. It has only cosine terms, and can be evaluated over a quarter of a period, from 0 to *π*/2. The function over this interval is -cos*t* + 2.5, with *ω*0 = 1 and *T* = 2*π*. It follows that:

*an* = ==

=

=

, where *n* has already been constrained by assuming quarter wave symmetry to have odd values only. For all odd values of *n* >1, sin(*n* ± 1)(*π*/2) = 0. For *n* = 1, sin(2)(*π*/2) = 0.  -sinc(*n* – 1)(*π*/2) = -1 when *n* = 1. It follows that *a*1 = -1 + 10/*π*, *a*3 = -10/3*π*, *a*5 = 10/5*π*, etc. The FSE is:



**P16.1.28** Derive the FSE of the waveform of Figure P16.1.28 in two ways: (a) direct evaluation of coefficients; (b) as the sum of two shifted rectangular pulse trains.



**Solution:** (a) *a*0 = , *T* = 4 s, *ω*0 = ;

*an* =  =

= [3sin*nω*0+ 2sin2*nω*0 − 2sin*nω*0 − 2sin4*nω*0 + 2sin2*nω*0] = [sin*nω*0+ 4sin2*nω*0 − 2sin4*nω*o] = [sin*nπ*/2+ 2sin*nπ*] = , for odd *n* and is zero for even *n*.

*bn* = 

=

=[3cos*nω*0– 3 + 2cos2*nω*0 – 2cos*nω*0 – 2cos4*nω*0 + 2cos2*nω*0]

=[cos*nω*0+ 4cos2*nω*0 – 2cos4*nω*0 – 3] = [cos*nπ*/2+ 4cos*nπ* – 2cos2*nπ* – 3] for odd *n,* and  for even *n*. It follows that: *f*(*t*) = + sin*πt* +  .

(b) From Equation 16.2.24 with *α* = ,



*A* = 1, and *t* replaced by *t* – 1/2, or *t* – *T*/8, so that *ω*0*t* is replaced by (*ω*0*t* – ), we have *f*1(*t*) =   = 

To this must be added *f*2(*t*), which is a square waveform of period 4 s and amplitude *A* = 2. From Equation 16.2.28: *f*2(*t*) = 

. Adding *f*1(*t*) and *f*2(*t*): *f*(*t*) = + sin*πt* + , as in (a).